Math 10A with Professor Stankova
Worksheet, Discussion #21; Friday, 10/13/2017
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True/False

1. **TRUE** False For $f(x)$ continuous on $[a, b]$, the function $F(x) = \int_a^x f(u)du$ is the unique anti-derivative of f on $[a, b]$ satisfying $F(a) = 0$.
2. **TRUE** False If $f(x)$ is continuous on $[a, b]$, then $F(x) = \int_a^x f(u)du$ is continuous on (a, b).
3. **TRUE** False We can use the addition differentiation law to prove the addition integration law for indefinite integrals $(\int(f + g) = (\int f) + (\int g))$.

Integration by Parts

Examples

4. True **FALSE** Integration by parts will automatically give the antiderivative of a function.
5. Find $\displaystyle\int \arctan(x)dx$.

Solution: For choosing a u, we should look for logarithms and inverse trig functions first. Then, we should look for polynomials, then trig functions and finally exponential functions. So, we set $u = \arctan(x)$ and set dv as the rest or $dv = 1dx$. Thus, we have that

$$\int \arctan(x)dx = uv - \int vdu = x\arctan(x) - \int \frac{x}{1 + x^2}dx.$$

We can solve the last one with a u substitution $u = 1+x^2$ so $du = 2xdx$ and $xdx = \frac{du}{2}$ so

$$\int \arctan(x)dx = x\arctan x - \frac{1}{2}\int \frac{du}{u} = x\arctan x - \frac{\ln(1 + x^2)}{2} + C.$$

6. Find $\int \sin(x)\cos(x)dx$.

Solution: We can use u substitution but also use integration by parts. Let $u = \sin(x)$ and $dv = \cos(x)dx$ so $v = \sin(x)$. Thus

$$\int \sin(x)\cos(x)dx = \sin^2(x) - \int \cos(x)\sin(x)dx.$$

So we have that

$$2\int \sin(x)\cos(x)dx = \sin^2(x) + C \implies \int \sin(x)\cos(x)dx = \frac{\sin^2(x)}{2} + C.$$

7. Integrate $\int 2x^3\cos(x^2)dx$.

Solution: Let $u = \cos(x^2)$ and $dv = 2x^3dx$ so $du = -\sin(x^2)\cdot 2xdx$ and $v = \frac{x^4}{2}$. Then

$$\int 2x^3\cos(x^2)dx = \frac{x^4\cos(x^2)}{2} - \int -\sin(x^2)x^5dx.$$

This hasn't been simplified at all so we should try another approach. Instead, first u sub to get $u = x^2$ and $dx = 2x$ so $2x^3dx = 2xdx(x^2) = udu$. Therefore, we get

$$\int 2x^3\cos(x^2)dx = \int u\cos(u)du = u\sin(u) + \cos(u) + C = x^2\sin(x^2) + \cos(x^2) + C.$$

Problems

8. Integrate $\int x\ln xdx$.

Solution: Let $u = \ln x$ and $dv = xdx$ so

$$\int x\ln xdx = \frac{x^2\ln x}{2} - \int \frac{x^2}{2}\frac{1}{x}dx = \frac{x^2\ln x}{2} - \int \frac{x}{2}dx = \frac{x^2\ln x}{2} - \frac{x^2}{4} + C.$$

9. Integrate $\int \frac{\ln x}{x^5}dx$.

Solution: Let $u = \ln x$ and $dv = x^{-5}dx$ so $v = \frac{-x^{-4}}{4}$ and so

$$\int \frac{\ln x}{x^5}dx = \frac{-\ln x}{4x^4} - \int \frac{-1}{4x^4} \cdot \frac{1}{x}dx = \frac{-\ln x}{4x^4} - \int \frac{-1}{4x^5}dx = \frac{-\ln x}{4x^4} - \frac{1}{16x^4} + C.$$

10. Integrate $\int 2x \arctan(x)dx$.

Solution: Let $u = \arctan(x)$ and $dv = 2xdx$ so $v = x^2$. Then

$$\int 2x \arctan(x)dx = x^2 \arctan(x) - \int \frac{x^2dx}{1+x^2} = x^2 \arctan(x) - \int \frac{x^2+1}{x^2+1} - \frac{1}{x^2+1}dx$$

$$= x^2 \arctan(x) - x + \arctan(x) + C.$$

11. Integrate $\int (\ln x)^2 dx$.

Solution: Let $u = (\ln x)^2 dx$ and $dv = 1dx$ so $v = x$. Then $du = \frac{2\ln x}{x}dx$ and

$$\int (\ln x)^2 dx = x(\ln x)^2 - \int \frac{2x \ln x}{x}dx = x(\ln x)^2 - 2\int \ln x dx$$

$$= x(\ln x)^2 - 2\left[x \ln x - \int x/xdx\right] = x(\ln x)^2 - 2x\ln x + 2x + C.$$

12. Integrate $\int x(\sin x + \cos x)dx$.

Solution: Let $u = x$ and $dv = (\sin(x) + \cos(x))dx$ so $du = dx$ and $v = -\cos(x) + \sin(x)$. So

$$\int x(\sin(x) + \cos(x))dx = x(\sin(x) - \cos(x)) - \int \sin(x) - \cos(x)dx$$

$$= x(\sin(x) - \cos(x)) - (-\cos(x) - \sin(x)) + C.$$

13. Integrate $\int \frac{\ln\sqrt{x}}{\sqrt{x}}dx$.

Solution: Let $u = \ln(\sqrt{x})$ and $dv = \frac{dx}{\sqrt{x}} = x^{-1/2}dx$ so $v = 2x^{1/2} = 2\sqrt{x}$. Then by chain rule, we have $du = \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}dx = \frac{1}{2x}dx$. Thus, we ahve

$$\int \frac{\ln\sqrt{x}}{\sqrt{x}}dx = 2\sqrt{x}\ln\sqrt{x} - \int 2\sqrt{x} \cdot \frac{dx}{2x} = 2\sqrt{x}\ln\sqrt{x} - \int \frac{dx}{\sqrt{x}} = 2\sqrt{x}\ln\sqrt{x} - 2\sqrt{x} + C.$$

Numerical Integration

14. True **FALSE** For calculating the error bound when using left endpoint method when approximating the integral of f on the interval $[a, b]$, we use $K_1 = f'(a)$.
15. **TRUE** False The error for an integral approximation can be negative.
16. True **FALSE** The error bound gives us what the exact error of using the different approximation techniques are.
17. True **FALSE** The error bounds aren't helpful because they don't give us the exact error.